Magnetic flux in mesoscopic rings: capacitance, inertia and kinetics

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We consider mesoscopic non-superconducting rings with an effective capacitance. We propose a Hamiltonian model describing magnetic flux in such rings. Next we incorporate dissipation and thermal fluctuations into our kinetic model. We consider kinetics in limiting regimes of strong and weak coupling to thermal bath.

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1. Introduction

It is more than one decade after the first experiments [1] proving the existence of the theoretically predicted [2, 3] persistent currents in normal metal multiply connected samples.

It has been expected [4] that currents in mesoscopic rings can flow even in the absence of any driving. Such *self-sustaining* currents although theoretically predicted have not been observed so far. One of the reasons is that precise experimental regimes for observation of such currents have not been suggested.

In our earlier work [5] we proposed the semi-phenomenological model of noisy dynamics of a magnetic flux produced by a current flowing in a mesoscopic ring or cylinder. In this model, the time evolution of the magnetic flux is governed by a classical Langevin equation analogical to the one for an overdamped Brownian particle in a specific potential.

In previous studies we have included both dissipation, related to the effective resistance of the ring, and thermal equilibrium fluctuations but the possible capacitance of the ring has been systematically neglected. The aim of this paper is to construct an extended version of the previous theory which takes into account effects related to the capacitance, which plays a role of the mass of a Brownian particle and is related to the inertia of the system.

First, in the section 2 we present the possible source of the capacitance of the mesoscopic ring and we formulate the effective Hamiltonian of the system. Next, in the section 3, we include terms which consistently describe dissipation processes. In the section 4, we consider characteristic time scales in the system, which in the limiting cases allow to simplify the evolution equation for the magnetic flux. We end with the summary in the section 5.

2. Hamiltonian model

Small disordered metallic rings and cylinders threaded by a magnetic flux display persistent, non-dissipative, currents run by coherent electrons. At finite temperature T some of the electrons become 'normal', i.e. non-coherent and the amplitude of the persistent current decreases. The dissipative motion of normal electrons is affected by thermal equilibrium Nyquist noise. We make an attempt to describe this situation by means of a two-fluid model, where normal and coherent electrons coexist.

We consider the ring as a set of N one dimensional current channels stacked along a certain axis. The coherent current as a function of magnetic flux depends on the parity of the number of coherent electrons in a channel. Let p denote the probability of an even number of coherent electrons in a single current channel. The formula for the coherent current reads [6]

$$I_{coh}(\phi, T) = pI_{even}(\phi, T) + (1-p)I_{odd}(\phi, T)$$

with

$$I_{even}(\phi, T) = I_{odd}(\phi - \phi_0/2, T) = I_0 \sum_{n=1}^{\infty} A_n(T) \sin(2n\pi\phi/\phi_0).$$

The flux quantum $\phi_0 = h/e$ and the maximal current $I_0 = heN_eN/(2l^2 m_e)$, where N_e is the number of coherent electrons in a single channel of a circumference l and m_e stands for the electron mass. The temperature dependent amplitudes read

$$A_n(T) = \frac{4T}{\pi T^*} \frac{\exp(-nT/T^*)}{1 - \exp(-2nT/T^*)} \cos(nk_F l),$$

where the characteristic temperature T^* is proportional to the energy gap Δ_F at the Fermi surface and k_F denotes the Fermi momentum. The energy gap Δ_F in metals occurs due to finite small size of the system.

We shall now construct an effective 'Hamiltonian' of the system. The total energy of the system consists of three parts. The first is the effective potential related to the persistent current itself; the second is related to the

energy of the magnetic flux and the third is due to charging effects caused by a small but non-zero capacitance C of the system.

The total energy of the set of discrete energy levels carrying persistent current can easily be related to this current at $T = 0$:

$$E_{coh}^{(0)}(\phi) = -\int I_{coh}(\phi, 0) d\phi. \tag{1}$$

At non-zero temperature T, the energy levels become blurred but they are still able to carry persistent currents as a suitable sum of single-level contributions weighted with the Fermi-Dirac probability distribution [6].

In the following we apply this approach and *define* the effective potential related to the persistent current by the relation

$$E_{coh}(\phi) = -\int I_{coh}(\phi, T) d\phi, \tag{2}$$

which reflects the well known fact that the persistent current is an equilibrium and thermodynamic rather than transport phenomenon. This approach is applicable as long as the quantum part of the system which is responsible for the persistent currents remains in equilibrium i.e. the notion of discrete energy levels makes sense. The high temperature limit does not violate the assumptions of the model due to vanishing amplitude of persistent currents. The approach used could be justified in a more elegant way applying the methods of thermofield dynamics [7].

In the following we assume that the ring possesses also an effective capacitance C. It was shown [8] that in the diffusive regime the energy associated with long-wavelength and low-energy charge fluctuations is determined by classical charging energies of suitable defined capacitors. The flux dependence of these energies yields the contribution to the persistent current. In other words, the ring behaves as it were a classical capasitor. The possibility that the charging energies could contribute to persistent current has been mentioned by Imry and Altshuler [9]. They claimed that the local charge fluctuations in a globally neutral system might be the key to understanding properties of persistent currents.

For the above mentioned three parts of the energy, the effective Hamiltonian takes the form

$$H = \frac{C}{2}\left(\frac{d\phi}{dt}\right)^2 + \frac{1}{2\mathcal{L}}(\phi - \phi_e)^2 + E_{coh}(\phi), \tag{3}$$

where ϕ_e is the magnetic flux induced by an external magnetic field B and $\mathcal{L}$ is a self-inductance of the system. Let us note that this Hamiltonian depends on temperature via the expression (2). It is not an exception because all mean-field Hamiltonians are temperature dependent. Moreover, it

can be interpreted in terms of thermofield dynamics [7]. The corresponding equation of motion reads

$$C\frac{d^2\phi}{dt^2} = -\frac{1}{\mathcal{L}}(\phi - \phi_e) + I_{coh}(\phi, T). \tag{4}$$

It describes a conservative system.

3. Capacitive model with dissipation

In real systems a dissipative processes take place at non-zero temperature. Therefore the resistance of the ring and thermal fluctuations should be taken into account. For $T > 0$, there are coherent and dissipative parts of the total current,

$$I_{tot} = I_{coh} + I_{dis}. \tag{5}$$

The dissipative current I_{dis} is determined by the Ohm's law and Lenz's rule,

$$I_{dis} = I_{dis}(\phi, T) = -\frac{1}{R}\frac{d\phi}{dt} + \sqrt{\frac{2k_BT}{R}}\,\Gamma(t)\ , \tag{6}$$

where R is the resistance of the ring and k_B denotes Boltzmann's constant. We model the thermal Nyquist fluctuations $\Gamma(t)$ of the Ohmic current by means of δ-correlated Gaussian white noise of zero average, i.e., $\langle\Gamma(t)\rangle = 0$ and $\langle\Gamma(t)\Gamma(s)\rangle = \delta(t-s)$. The noise intensity $D_0 = 2k_BT/R$ is chosen in accordance with the fluctuation-dissipation theorem.

Now, we generalize Eq. (4) by replacing I_{coh} by the total current I_{tot}. We obtain then the basic evolution equation

$$C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} = -\frac{1}{\mathcal{L}}(\phi - \phi_e) + I_{coh}(\phi, T) + \sqrt{\frac{2k_BT}{R}}\,\Gamma(t). \tag{7}$$

By including the 'inertial' capacitive term we get an extended version of the equation proposed in [5].

4. Overdamped and underdamped regimes

It is useful to work in dimensionless variables, because relations between scales of time, energy, currents, fluxes, etc., play a crucial role. The characteristic magnetic flux is determined in a natural way by the flux quantum $\phi_0 = h/e$. Accordingly, the flux is scaled as $x = \phi/\phi_0$. Time can be scaled in several ways. In the following two examples we recognize some of the possible time scales and corresponding energy scales of the system (7).

4.1. Overdamped limit

The capacitance of ideally pure rings can safely be neglected and hence the second order term in the equation of motion (7) for the magnetic flux can also be neglected. If the damping effects are dominating then the characteristic time τ_0 can be obtained from the equation

$$\frac{1}{R}\frac{d\phi}{dt} = -\frac{1}{\mathcal{L}}(\phi - \phi_e) \tag{8}$$

by inserting the characteristic quantities, namely,

$$\frac{1}{R}\frac{\phi_0}{\tau_0} = \frac{\phi_0}{\mathcal{L}}, \quad \text{or} \quad \tau_0 = \frac{\mathcal{L}}{R}. \tag{9}$$

In the dimensionless units, for the rescaled flux $x = \phi/\phi_0$ and rescaled time $\tau = t/\tau_0$, Eq. (7) can be rewritten in dimensionless form as

$$\mathcal{M}\frac{d^2x}{d\tau^2} + \frac{dx}{d\tau} = -\frac{dV(x)}{dx} + \sqrt{2D}\,\xi(\tau). \tag{10}$$

The rescaled noise intensity is $D = k_B T/2\varepsilon_0$, the characteristic energy is $\varepsilon_0 = \phi_0^2/2\mathcal{L}$, the external flux scales as $x_e = \phi_e/\phi_0$. The generalized potential reads

$$V(x) = \frac{1}{2}(x - x_e)^2 + F(x) \tag{11}$$

with

$$F(x) = \alpha \sum_{n=1}^{\infty} \frac{A_n(T)}{2n\pi} \left\{ p\cos(2n\pi x) + (1-p)\cos\left[2n\pi(x + 1/2)\right] \right\}, \tag{12}$$

where $\alpha = \mathcal{L}I_0/\phi_0$.

The rescaled zero-mean Gaussian white noise $\xi(\tau)$ has the same statistical properties as the thermal noise $\Gamma(t)$. Eq. (10) corresponds to the evolution equation for the position x of a Brownian particle of mass $\mathcal{M}$ evolving in a potential $V(x)$. The charging (inertial) effects are characterized by the rescaled 'mass'

$$\mathcal{M} = \frac{CR^2}{\mathcal{L}} = \frac{\tau_L}{\tau_0}, \tag{13}$$

which is the ratio of the two characteristic times $\tau_L = CR$ and $\tau_0 = \mathcal{L}/R$.

The Fokker-Planck equation for the probability distribution $p(x, \dot{x}, t)$ (with $\dot{x} = dx/d\tau$) corresponding to (10) takes the form of a Kramers-Klein

equation for an inertial Brownian particle. Its stationary solution $p_{st}(x, \dot{x})$ is the Gibbs distribution describing the equilibrium state, namely,

$$p_{st}(x, \dot{x}) \propto \exp\left\{-\frac{1}{D}\left[\frac{\mathcal{M}\dot{x}^2}{2} + V(x)\right]\right\}. \tag{14}$$

When the charging effects can be neglected, i.e. when the 'mass' $\mathcal{M} \ll 1$ we recover the overdamped (Smoluchowski) limit. In such a case Eq. (10) reduces to the form

$$\frac{dx}{d\tau} = -\frac{dV(x)}{dx} + \sqrt{2D}\,\xi(\tau). \tag{15}$$

The stationary distribution $P_{st}(x)$ in the projected 'coordinate' space x can be obtained from (14) by integration of the distribution $p(x, \dot{x})$ over the 'velocity' variable $\dot{x}$

$$P_{st}(x) \propto \exp\left[-V(x)/D\right]. \tag{16}$$

The Smoluchowski regime has been studied in [5].

4.2. Underdamped limit

If the 'inertial' (charging) effects dominate, then the characteristic time τ_1 can be obtained from the relation

$$C\frac{d^2\phi}{dt^2} = -\frac{1}{\mathcal{L}}(\phi - \phi_e) \tag{17}$$

by inserting the characteristic quantities,

$$C\frac{\phi_0}{\tau_1^2} = \frac{\phi_0}{\mathcal{L}} \quad \text{or} \quad \tau_1^2 = \mathcal{L}C. \tag{18}$$

In the scaling $\tau = t/\tau_1$, Eq. (7) assumes the form

$$\frac{d^2x}{d\tau^2} + \eta\frac{dx}{d\tau} = -\frac{dV(x)}{dx} + \sqrt{2D_1}\,\xi(\tau). \tag{19}$$

The rescaled friction coefficient η is the ratio of two characteristic times, namely,

$$\eta = \frac{\tau_0}{\tau_1}. \tag{20}$$

The rescaled noise intensity is $D_1 = \mathcal{L}^2 k_B T/\phi_0^2 R\tau_1$. In the underdamped limit, the friction coefficient $\eta \ll 1$. Then Eq. (19) reduces to the form

$$\frac{d^2x}{d\tau^2} = -\frac{dV(x)}{dx} + \sqrt{2D_1}\,\xi(\tau). \tag{21}$$

In the noiseless case, when $\xi(\tau) = 0$, this system is conservative.

There are also other time scales and there is no unique prescription which scaling is better. If there is an evident time scale separation (as e.g. $\mathcal{M} \ll 1$ or $\eta \ll 1$), then one can eliminate the corresponding fast variables and get the effective evolution equations for the slow variables. This method is well-known as the method of adiabatic elimination of fast variables and has been exploited for decades. The above two equations (15) and (21) are examples of this method.

5. Summary

In this paper we relate the effective capacitance of mesoscopic ring to the inertial effects in the kinetics of magnetic flux. We show the possible source of such an effective capacitance. For such systems, we formulate the Hamiltonian and the corresponding equation of motion. Contrary to prior studies, the proposed model includes also a second order term, which describes the inertial effects.

The kinetics of a real system under influence of dissipation and fluctuations is formulated. The proper form of the noise term ensures that the equilibrium conditions are satisfied. After identification of the characteristic time scales we consider the limiting cases of under- and overdamped systems.

There is a natural, but formal, analogy between the model introduced in this paper and that of noisy kinetics of the magnetic flux in a superconducting ring with a Josephson junction [10]. In both cases one deals with a non-linear oscillator subjected to non-linear forces. Even at this formal level there is a difference since in the case of normal metal rings the force term depends on temperature.

It is known that Josephson junctions with a large charging energy can exhibit quantum mechanical properties: they can operate in the so-called quantum Smoluchowski regime [11]. In the case of non-superconducting rings the limit of small capacity is natural and quantum effects are expected to be more pronounce.

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